Lateral Range Curves, Search Probabilities, and Grid Searching

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Abstract

The Lateral Range Curve is a fundamental component of search theory, but its importance is not always appreciated and it is often misunderstood. This paper looks at two lateral range curves in some detail, and highlights the difference between the probability of the searcher detecting a search object as demonstrated by the lateral range curve, and the probability of the searcher detecting a search object in the generally accepted meaning of the term "probability of detection", POD.

It goes on to consider the problem of grid searching, and shows how each of the lateral range curves gives rise to a distinct and different graph of POD as a function of Coverage.

The paper is of necessity mathematical in nature, but there is sufficient non-mathematical content to put the principles within the reach of most land SAR searchers, and to provide them with a good basis for understanding the application of search theory to land SAR.

An Introduction to Lateral Range Curves

Imagine a searcher following an infinitely long, straight path, searching on either side of that path. That searcher's Lateral Range Curve $p(x)$ is the probability of detecting a stationary object that is **at its closest exactly** a distance x from the searcher's path on the ground.[1] Note, this is not the usual probability of detection (POD), which is the probability of detecting an object that is **within** a distance x of the searcher's path. This distinction has been not been made by some authors and has lead to confusion.[2]

By convention, the searcher's path is assumed to be $x = 0$, with $x > 0$ being to the searcher's right, and $x < 0$ to the searcher's left.

There are an infinite number of possible Lateral Range Curves (LRCs), three examples of which follow.[3] In these examples, the quantity M is a positive constant.

- **The Definite Range Lateral Range Curve Model**

$$p(x) = \begin{cases} 1 & \text{if} \quad -M/2 \le x \le M/2, \\ 0 & \text{if} \quad |x| > M/2. \end{cases}$$

[1] In the case of an aircraft, x, is measured from the object to the projection of the flight-path on the ground.

[2] See, for example, "Overview of Search Theory, Part 1" by Lee Lang, *Technical Rescue*, Issue 50, 2007, page 30.

[3] See also "Search and Screening" by Bernard O. Koopman, MORS, Virginia, 1999, page 64, and "Search and Detection" by Alan R. Washburn, INFORMS, Maryland, 4th edition, 2002, Chapter 4.

Where it is not zero, this is a rectangle of height 1 and base-length M.

- **The Linear Lateral Range Curve Model**

$$p(x) = \begin{cases} 1 - \frac{|x|}{M} & \text{if} \quad -M \le x \le M, \\ 0 & \text{if} \quad |x| > M. \end{cases}$$

Where it is not zero, this is an isosceles triangle with height 1 and base length $2M$.

- **The Inverse Cube Lateral Range Curve Model**

$$p(x) = \begin{cases} 1 - e^{-M^2/(4\pi x^2)} & \text{if} \quad x \ne 0, \\ 1 & \text{if} \quad x = 0. \end{cases}$$

This is a bell-shaped curve that is never zero.

Figure 1 shows these functions, with $M = 1$.

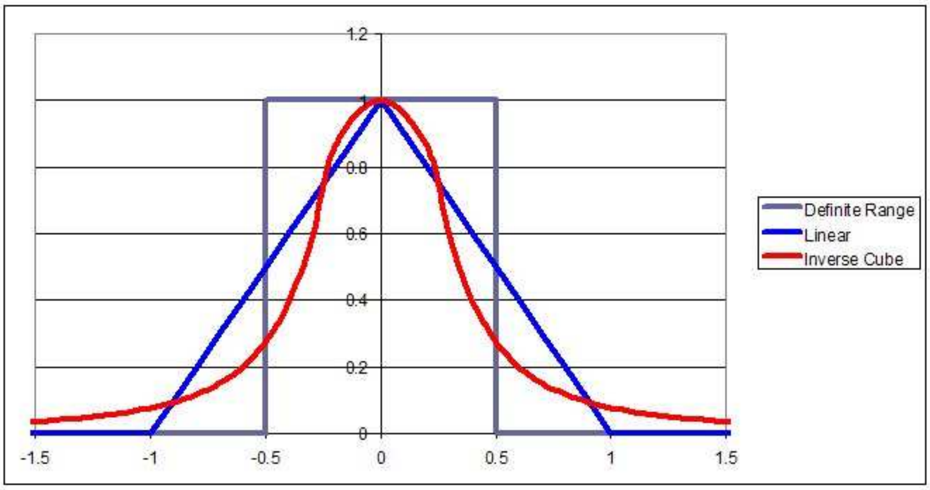

Figure 1: Lateral Range Curves

Each of these LRCs has the property that $p(0) = 1$, that is, for each of these models the probability of detecting an object that is on the searcher's path is 100%. While this may seem a reasonable property to expect of an LRC this is not be true for all situations. For example, the flight-path of a helicopter is unsearched if the pilot and co-pilot are concentrating on navigating and are not searching, while the searchers are looking out of the cargo doors and searching at right-angles to the flight-path. (This can occur when a "Huey" is used as the search platform.) Under these circumstances we have $p(0) = 0$, and none of the previous models would be useful in this case. Thus, in general, $p(0) \ne 1$.

Each of these LRCs has the property that they are symmetric about the vertical axis, that is $p(x) = p(-x)$. This means that the probability of detecting an object at distance x from the searcher's path is the same as the probability of detecting an object at distance $-x$ from the searcher's path. That is, the probability is the same to the right and left of the searcher's path. While this may seem a reasonable property to expect of an

LRC this will not be true for all situations. For example, if the searcher's path has a forest to the left and a meadow to the right, then you would not expect $p(x) = p(-x)$. The same is true for an air-scenting dog as it works across the wind trying to pick up the scent being carried downwind from a human scent source. The dog is much more likely to pick up the scent of a person who is 100 metres upwind of their path compared with 100 metres downwind of it; the dog's LRC would not be symmetrical. Thus, in general, $p(x) \neq p(-x)$.

A LRC is a characteristic of a sensor, searching for a particular object in a particular environment. A change in any of these will bring about a change in the shape of the LRC. For example, if we replace the search objects in the original example with a different set of objects that are larger and in a high visibility colour, then, because they can be seen at a greater distance than the original objects, the three LRCs in Fig. 1 would extend out farther than they currently do.

Properties of LRCs

All LRCs share the following properties.

1. $p(x)$ is defined for all x.

2. $p(x)$ is a probability, so $0 \leq p(x) \leq 1$.

3. The area between the $p(x)$ and the x-axis is finite, so that $\int_{-\infty}^{\infty} p(x)\,dx$ converges. This area, W, is called the **Effective Sweep Width**, so

$$W = \int_{-\infty}^{\infty} p(x)\,dx.$$

   In each of the previous three LRCs,

$$W = M,$$

   where W is the area under the LRC. This can be shown quite easily for the Definite Range and Linear LRCs in Fig. 1 (remember that $M = 1$); the Inverse Cube Model could be dealt with in the same way but is more complicated

4. Functions whose graphs have vertical sides are allowed. This is accomplished if $p(x)$ is either continuous or has a finite number of jump discontinuities.[4]

From now on we will concentrate on the Definite Range LRC model and the Linear LRC model.

Grid Searching and Coverage

In land SAR, ground searchers generally search not on their own but as part of an organized group. However, an aircraft involved in a land search is usually searching on its own, and will most likely be flying a search pattern called a creeping line; that is beyond the scope of the current document and will be dealt with in a later paper.

The basic method of searching areas in the later stages of a land search for a missing person is grid searching. In grid searching, the searchers in a field team form a line with equal spacing between adjacent searchers at the start of the area they are to search. This has the effect of defining, on the ground ahead of

[4]A jump discontinuity of a function $f(x)$ is a point a where the limit of $f(x)$ as x approaches a from the left and the corresponding limit from the right both exist but are distinct.

each searcher, a strip of ground that is their responsibility to search. The center line of this strip of ground and its projection forward through the search area represents the ideal path along which the searcher would move. The search field team then moves forward together, maintaining the original spacing. The searchers are moving forward together in a series of regularly spaced parallel paths.

We let the track spacing (the distance between the parallel paths) be nW, where W is the effective sweep width and $n > 0$. The coverage, C, is calculated as the ratio of the search effort to the area of the sector being searched, which is

$$C = \frac{\text{Total distance travelled by the searchers} \times \text{ Effective sweep width}}{\text{Area of the sector}},$$

and it can be easily shown that this is equivalent to

$$\begin{aligned} C &= \frac{\text{Effective sweep width}}{\text{Track spacing}} \\ &= \tfrac{W}{nW} \\ &= \tfrac{1}{n}. \end{aligned}$$

In other words, as the spacing decreases (n gets smaller) and the searchers get closer together then the coverage C increases; as the spacing increases (n gets larger) and the searchers get further apart then the coverage C decreases.

In the analysis that follows, we will examine the effect that varying the spacing between adjacent searchers has on the POD of a line of grid searchers. We will consider searcher spacings from wide apart ($n > 1$ and $C < 1$) to close together ($n = \frac{1}{2}$ and $C = 2$). We will do this for both the Definite Range LRC and the Linear LRC. The analysis assumes that all of the searchers are at the same spacing, and that the spacing remains constant throughout. It also assumes that the search object is between the searchers.

Grid Search: Definite Range LRC Model

A field team consists of a number of searchers operating together. Each searcher can be represented by a lateral range curve. The method used to calculate their POD is to place a copy of the lateral range curve for each searcher next to each other at a common distance nW apart.

The formula $C = 1/n$ gives the coverage for this distance and therefore the POD can be determined for various values of coverage.

Figures 2 and 3 represent the two different situations that need to be considered regarding the spacing between two searchers.

Figure 2 represents two searchers separated by a distance nW where $n \geq 1$. Here the searchers are sufficiently far apart for there to be no overlap of the lateral range curves. This means that, depending on its position, there is a chance that an object between the two searchers will be seen by either of them or by neither of them. It is not possible for both of them to see it.

Figure 3 represents two searchers separated by a distance nW where $n \leq 1$. Here the lateral range curves overlap in the area between the searchers. In this case, depending on its position, there is a chance that one or both of the searchers could see the search object that is situated in the region between them.

In Fig. 2 the coverage C is in the range $0 < C \leq 1$; in Fig. 3 it is in the range $C \geq 1$. This provides a useful check on the calculation process in that the POD for the end values for the ranges ($C = 1$) should be the same whichever range has been considered.

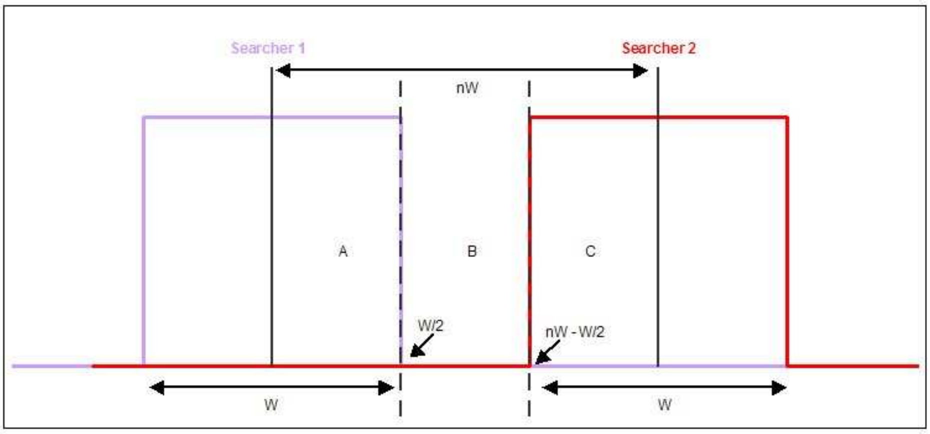

Figure 2: Two searchers separated by nW, where $n \geq 1$

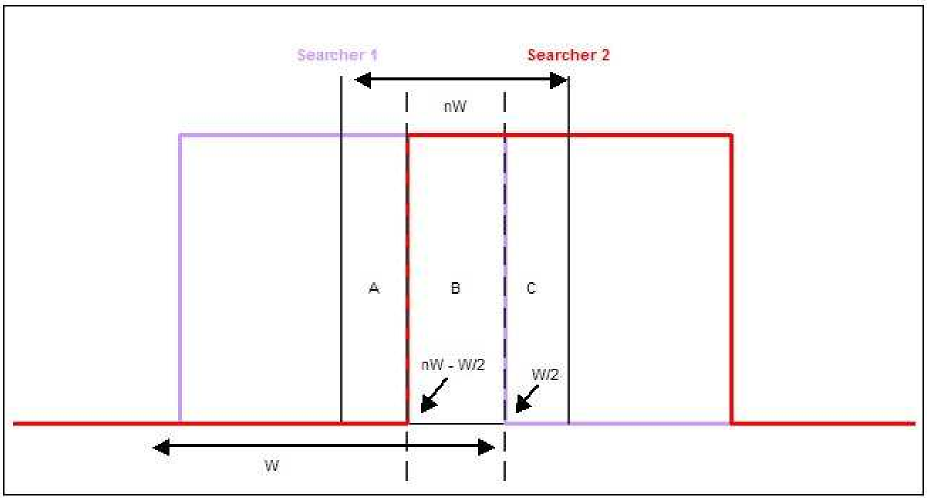

Figure 3: Two searchers separated by nW, where $0.5 \leq n \leq 1$

If $p_k(x)$ represents the probability that an object at distance x is seen by searcher k, and $\widetilde{p_k}(x) = 1 - p_k(x)$ the probability that it is not seen by searcher k then the probability $P(x)$ that this object is seen by either searcher 1 or by searcher 2 or by both is given by

$$P(x) = 1 - \widetilde{p_1}(x)\widetilde{p_2}(x).$$

Then the probability of seeing an object located anywhere in a rectangular region of height 1 whose base starts at $x = a$ and ends at $x = b$ (where $a < b$), is given by

$$\int_a^b P(x)\,dx.$$

POD for Coverage ≤ 1

In Fig. 2 we see that there are three regions to consider, A, B, and C.

For an object located in region A at a distance x from searcher 1 we have $0 \leq x \leq W/2$, so $p_1(x) = 1$ and $\widetilde{p_1}(x) = 1 - p_1(x) = 0$. We also have $p_2(x) = 0$ and $\widetilde{p_2}(x) = 1 - p_2(x) = 1$, so that the probability that the object at a distance x from searcher 1 is seen is

$$P(x) = 1 - \widetilde{p_1}(x)\widetilde{p_2}(x) = 1.$$

Thus, the probability of seeing an object located anywhere in region A is

$$P_A = \int_0^{W/2} P(x)\,dx = \int_0^{W/2} 1\,dx = \frac{W}{2}.$$

In the same way, we find that the probability of seeing an object located anywhere in region B is $P_B = 0$, and the probability of seeing an object located anywhere in region C is $P_C = W/2$. Notice that $P_A = P_C$, as expected by symmetry.

Thus, the probability of detection for an object located anywhere between searcher 1 and searcher 2 in Fig. 2 is

$$POD = \frac{\text{Total number of objects that are found}}{\text{Total number of objects that are available to be found}}.$$

The numerator is the sum of the probabilities for the three regions A, B and C, which is equivalent to the total number of objects found between the two searchers; the denominator is $nW \times 1$, which is the area of the rectangle in Fig. 2 between the searchers that contains all of the search objects. Thus,

$$POD = \frac{P_A + P_B + P_C}{nW} = \frac{W/2 + 0 + W/2}{nW} = \frac{1}{n} = C.$$

POD for Coverage $1 \leq C \leq 2$

In Fig. 3 we see that there are again three regions to consider, A, B, and C.

For an object located in region A at a distance x from searcher 1 we have $0 \leq x \leq nW - W/2$, so $p_1(x) = 1$ and $\widetilde{p_1}(x) = 1 - p_1(x) = 0$. We also have $p_2(x) = 0$ and $\widetilde{p_2}(x) = 1 - p_2(x) = 1$, so that

$$P(x) = 1 - \widetilde{p_1}(x)\widetilde{p_2}(x) = 1.$$

Thus, the probability of seeing an object located anywhere in region A is

$$P_A = \int_0^{nW-W/2} 1\,dx = nW - \frac{W}{2}.$$

For an object located in region B at a distance x from searcher 1 we have $nW - W/2 \le x \le W/2$, so $p_1(x) = 1$ and $\widetilde{p_1}(x) = 1 - p_1(x) = 0$. We also have $p_2(x) = 1$ and $\widetilde{p_2}(x) = 1 - p_2(x) = 0$, so that

$$P(x) = 1 - \widetilde{p_1}(x)\widetilde{p_2}(x) = 1.$$

Thus, the probability of seeing an object located anywhere in region B is

$$P_B = \int_{nW-W/2}^{W/2} 1\,dx = \frac{W}{2} - \left(nW - \frac{W}{2}\right) = W - nW.$$

For an object located in region C at a distance x from searcher 1 we have $W/2 \le x \le nW$, so $p_1(x) = 0$ and $\widetilde{p_1}(x) = 1 - p_1(x) = 1$. We also have $p_2(x) = 1$ and $\widetilde{p_2}(x) = 1 - p_2(x) = 0$, so that

$$P(x) = 1 - \widetilde{p_1}(x)\widetilde{p_2}(x) = 1.$$

Thus, the probability of seeing an object located anywhere in region C is

$$P_C = \int_{W/2}^{nW} 1\,dx = nW - \frac{W}{2}.$$

Notice that $P_A = P_C$, as expected by symmetry.

Thus, the probability of detection for an object located anywhere between searcher 1 and searcher 2 in Fig. 3 is

$$POD = \frac{P_A + P_B + P_C}{nW} = \frac{(nW - W/2) + (W - nW) + (nW - W/2)}{nW} = 1.$$

Thus, we have

$$POD(C) = \begin{cases} 1 & \text{if} \quad 1 \le C \le 2, \\ C & \text{if} \quad 0 < C \le 1. \end{cases}$$

As a check, we see that when $C = 1$ then $POD(C) = 1$ in both cases.

Grid Search: Linear LRC Model

We now repeat this process for the Linear LRC Model.

A field team consists of a number of searchers operating together. Each searcher can be represented by a lateral range curve. The method used to calculate their POD is to place a copy of the lateral range curve for each searcher next to each other at a common distance nW apart. Thus, searcher 1's lateral range curve intersects the horizontal axis at $-W$ and W. Searcher 2 is immediately to searcher 1's right a distance nW from searcher 1, so searcher 2's lateral range curve intersects the horizontal axis at $nW - W$ and $nW + W$. Searcher 3 is immediately to searcher 2's right a distance $2nW$ from searcher 1, so searcher 3's lateral range curve intersects the horizontal axis at $2nW - W$ and $2nW + W$. And so on. In the same way, Searcher 0 is immediately to searcher 1's left a distance nW from searcher 1, so searcher 0's lateral range curve intersects the horizontal axis at $-nW - W$ and $-nW + W$.

Notice that although we will be following a similar approach to that used for the Definite Range model, for some of the time we will have to take into account searchers on either side of the two searchers on whom our attention is focused. This is because when coverage $C > 1$, the spacing between adjacent searchers is less than W, and consequently there is a chance that these outer searchers could detect an object in the space that we are examining.

There are various possibilities that occur as n decreases, and more and more of these lateral range curves overlap. Each of the four cases that follow is described in terms of the relationships between the points where the LRCs meet the horizontal axis in the space between searcher 1 and searcher 2. Start with the first point that occurs to the right of searcher 1 in the appropriate diagram, work towards and finish with searcher 2; remember that searcher 2 is always at a distance nW from searcher 1.

1. For large n, none of the lateral range curves will intersect, which occurs when the right-hand intersection of searcher 1's lateral range curve is less than the left-hand intersection of searcher 2's lateral range curve. This occurs when $W \leq W(n-1)$, that is, $n \geq 2$ or $C \leq 0.5$.

   Figure 4 represents two searchers separated by a distance nW where $n \geq 2$. Here the searchers are sufficiently far apart for there to be no overlap of the lateral range curves. This means that, depending on its position, there is a chance that an object between the two searchers will be seen by either of them or by neither of them. It is not possible for both of them to see it.

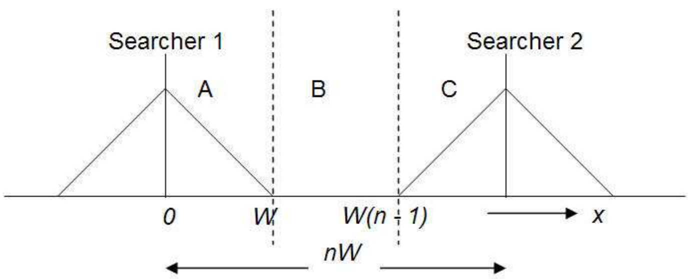

Figure 4: Two searchers separated by nW, where $n \geq 2$

2. The next change occurs when searcher 2's left-hand intersection is inside searcher 1's right-hand intersection, but searcher 3's left-hand intersection is not inside searcher 1's right-hand intersection. This occurs when $W(n-1) \leq W \leq nW$, that is, $1 \leq n \leq 2$ or $0.5 \leq C \leq 1$.

   Figure 5 represents two searchers separated by a distance nW where $1 \leq n \leq 2$. Here the lateral range curves overlap in the area between the searchers. In this case, depending on its position, there is a chance that one or both of the searchers could see the search object that is situated in the region between them.

3. The next change occurs when searcher 3's left-hand intersection is inside searcher 1's right-hand intersection, but is not inside searcher 0's right-hand intersection. This occurs when $(1-n)W \leq (2n-1)W \leq nW$, that is, $2/3 \leq n \leq 1$ or $1 \leq C \leq 1.5$.

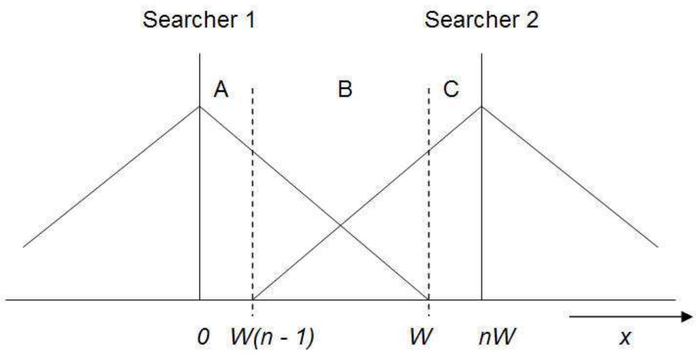

Figure 5: Two searchers separated by nW, where $1 \leq n \leq 2$

Figure 6 represents four searchers separated by a distance nW where $2/3 \leq n \leq 1$. Here the searchers are sufficiently close not only for their lateral range curves to overlap but there is now a chance that searchers 0 and 3 will see the search object in the region between searchers 1 and 2.

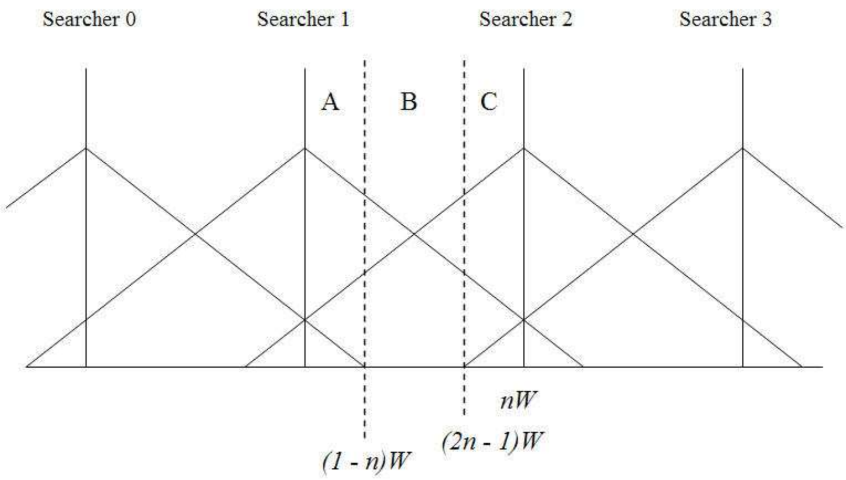

Figure 6: Four searchers separated by nW, where $2/3 \leq n \leq 1$

4. The final case occurs when searcher 3's left-hand intersection is inside searcher 0's right hand intersection. This occurs when $(2n-1)W \leq (1-n)W \leq nW$, that is, $1/2 \leq n \leq 2/3$, or $1.5 \leq C \leq 2$.

   Figure 7 represents four searchers separated by a distance nW where $1/2 \leq n \leq 2/3$. Here the searchers are sufficiently close not only for their lateral range curves to overlap but there is now a chance that that searchers 0 and 3 will see the search object in the region between searchers 1 and 2.

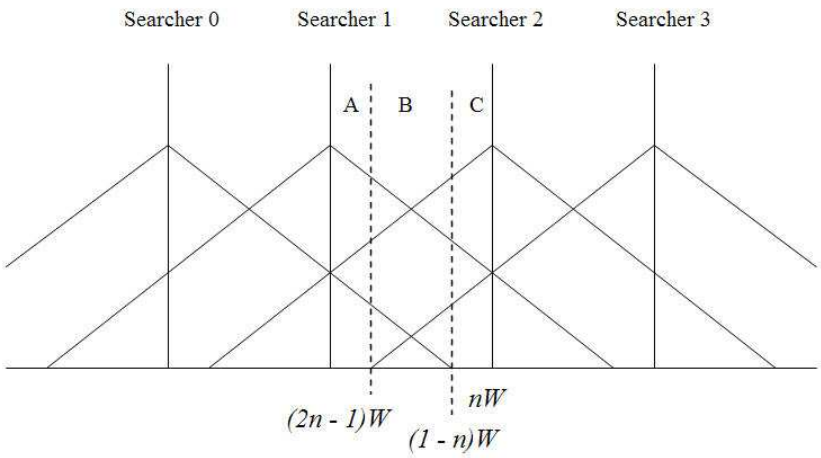

Figure 7: Four searchers separated by nW, where $1/2 \le n \le 2/3$

The procedure that we will use to determine the POD for a grid search team is to take each of the four cases that we have just identified, and in each case calculate the POD for the area between searchers 1 and 2. This will give the POD between any two adjacent searchers, and therefore by extension give the POD anywhere between the searchers at either end of the line. The corresponding results for the regions to the left of left-most searcher and to the right of right-most searcher require a different analysis, and will be dealt with in a later paper.

POD for Coverage ≤ 0.5

In Fig. 4 we see that there are three regions to consider, A, B, and C.

For an object located in region A at a distance x from searcher 1 we have $0 \le x \le W$, so $p_1(x) = 1 - x/W$ and $\widetilde{p_1}(x) = 1 - p_1(x) = x/W$. We also have $p_2(x) = 0$ and $\widetilde{p_2}(x) = 1 - p_2(x) = 1$, so that the probability that the object at a distance x from searcher 1 is seen is

$$P(x) = 1 - \widetilde{p_1}(x)\widetilde{p_2}(x) = 1 - \frac{x}{W}.$$

Thus, the probability of seeing an object located anywhere in region A is

$$P_A = \int_0^W \left(1 - \frac{x}{W}\right) dx = \frac{W}{2}.$$

For an object located in region B at a distance x from searcher 1, we have $W \le x \le (n-1)W$, so $p_1(x) = 0$ and $\widetilde{p_1}(x) = 1 - p_1(x) = 1$. We also have $p_2(x) = 0$ and $\widetilde{p_2}(x) = 1$. Thus, the probability of seeing an object located anywhere in region B is

$$P_B = \int_W^{(n-1)W} (1-1)\, dx = 0.$$

For an object located in region C at a distance x from searcher 1, we have $(n-1)W \le x \le nW$, so $p_1(x) = 0$ and $p_2(x) = 1 - (nW - x)/W$, giving $\widetilde{p_1}(x) = 1$ and $\widetilde{p_2}(x) = (nW - x)/W$. Thus, the probability of seeing an object located anywhere in region C is

$$P_C = \int_{(n-1)W}^{nW} \left(1 - \frac{(nW - x)}{W}\right) dx = \frac{W}{2}.$$

Notice that $P_A = P_C$, as expected by symmetry.

Thus, the probability of detection for an object located anywhere between searcher 1 and searcher 2 in Fig. 4 is

$$POD = \frac{P_A + P_B + P_C}{nW} = \frac{W/2 + 0 + W/2}{nW} = \frac{1}{n} = C.$$

POD for Coverage $0.5 \le C \le 1$

In Fig. 5 we see that there are again three regions to consider, A, B, and C.

For an object located in region A at a distance x from searcher 1 we have $0 \le x \le (n-1)W$, so $p_1(x) = 1 - x/W$ and $\widetilde{p_1}(x) = 1 - p_1(x) = x/W$. We also have $p_2(x) = 0$ and $\widetilde{p_2}(x) = 1 - p_2(x) = 1$, so that

$$P(x) = 1 - \widetilde{p_1}(x)\widetilde{p_2}(x) = 1 - \frac{x}{W}.$$

Thus, the probability of seeing an object located anywhere in region A is

$$P_A = \int_0^{(n-1)W} \left(1 - \frac{x}{W}\right) dx = \frac{W}{2}(4n - n^2 - 3).$$

For an object located in region B at a distance x from searcher 1, we have $(n-1)W \le x \le W$, so $p_1(x) = 1 - x/W$ and $\widetilde{p_1}(x) = 1 - p_1(x) = x/W$. We also have $p_2(x) = 1 - (nW - x)/W$ and $\widetilde{p_2}(x) = (nW - x)/W$, so that

$$P(x) = 1 - \widetilde{p_1}(x)\widetilde{p_2}(x) = 1 - \frac{x}{W}\frac{nW - x}{W} = \frac{1}{W^2}(W^2 - nWx + x^2).$$

Thus, the probability of seeing an object located anywhere in region B is

$$P_B = \frac{1}{W^2}\int_{(n-1)W}^{W} (W^2 - nWx + x^2)\, dx = \frac{W}{6}(n^3 - 12n + 16).$$

For an object located in region C at a distance x from searcher 1, we have $W \le x \le nW$, so $p_1(x) = 0$ and $p_2(x) = 1 - (nW - x)/W$, giving $\widetilde{p_1}(x) = 1$ and $\widetilde{p_2}(x) = (nW - x)/W$. Thus, the probability of seeing an object located anywhere in region C is

$$P_C = \int_W^{nW} \left(1 - \frac{(nW - x)}{W}\right) dx = \frac{W}{2}(4n - n^2 - 3).$$

Notice that $P_A = P_C$, as expected by symmetry.

Thus, the probability of detection for an object located anywhere between searcher 1 and searcher 2 in Fig. 5 is

$$POD = \frac{P_A + P_B + P_C}{nW} = \frac{2\frac{W}{2}(4n - n^2 - 3) + \frac{W}{6}(n^3 - 12n + 16)}{nW},$$

which reduces to

$$POD = \frac{n^3 - 6n^2 + 12n - 2}{6n} = \frac{1 - 6C + 12C^2 - 2C^3}{6C^2}.$$

POD for Coverage $1 \le C \le 1.5$

In Fig. 6 we see that, if we concentrate on the area between searchers 1 and 2, there are again three regions to consider, A, B, and C.

For an object located in region A at a distance x from searcher 1 we have $0 \le x \le (1-n)W$, so $p_0(x) = 1 - (nW + x)/W$, $p_1(x) = 1 - x/W$, $p_2(x) = 1 - (nW - x)/W$ and $p_3(x) = 0$. Thus,

$$P(x) = 1 - \widetilde{p_0}(x)\widetilde{p_1}(x)\widetilde{p_2}(x)\widetilde{p_3}(x) = 1 - \frac{nW + x}{W}\frac{x}{W}\frac{nW - x}{W} = \frac{W^3 - n^2W^2x + x^3}{W^3}.$$

Thus, the probability of seeing an object located anywhere in region A is

$$P_A = \frac{1}{W^3}\int_0^{(1-n)W} (W^3 - n^2W^2x + x^3)\,dx = \frac{W}{4}(5 - 8n + 4n^2 - n^4).$$

For an object located in region B at a distance x from searcher 1 we have $(1-n)W \le x \le (2n-1)W$, so $p_0(x) = 0$, $p_1(x) = 1 - x/W$, $p_2(x) = 1 - (nW - x)/W$ and $p_3(x) = 0$. Thus,

$$P(x) = 1 - \widetilde{p_0}(x)\widetilde{p_1}(x)\widetilde{p_2}(x)\widetilde{p_3}(x) = 1 - \frac{x}{W}\frac{nW - x}{W} = \frac{W^2 - Wnx + x^2}{W^2}.$$

Thus, the probability of seeing an object located anywhere in region B is

$$P_B = \frac{1}{W^2}\int_{(1-n)W}^{(2n-1)W} (W^2 - Wnx + x^2)\,dx = \frac{W}{6}(9n^3 - 24n^2 + 36n - 16).$$

For an object located in region C at a distance x from searcher 1 we have $(2n-1)W \le x \le nW$, so $p_0(x) = 0$, $p_1(x) = 1 - x/W$, $p_2(x) = 1 - (nW - x)/W$ and $p_3(x) = 1 - (2nW - x)/W$.

$$P(x) = 1 - \widetilde{p_0}(x)\widetilde{p_1}(x)\widetilde{p_2}(x)\widetilde{p_3}(x) = 1 - \frac{x}{W}\frac{nW - x}{W}\frac{2nW - x}{W} = \frac{W^3 - 2xn^2W^2 + 3x^2nW - x^3}{W^3}.$$

Thus, the probability of seeing an object located anywhere in region C is

$$P_C = \frac{1}{W^3}\int_{(2n-1)W}^{nW} (W^3 - 2xn^2W^2 + 3x^2nW - x^3)\,dx = \frac{W}{4}(5 - 8n + 4n^2 - n^4).$$

Notice that $P_A = P_C$, as expected by symmetry.

Thus, the probability of detection for an object located anywhere between searcher 1 and searcher 2 in Fig. 6 is

$$POD = \frac{P_A + P_B + P_C}{nW} = \frac{2\frac{W}{4}(5 - 8n + 4n^2 - n^4) + \frac{W}{6}(9n^3 - 24n^2 + 36n - 16)}{nW},$$

which reduces to

$$POD = \frac{-1 + 12n - 12n^2 + 9n^3 - 3n^4}{6n} = \frac{-3 + 9C - 12C^2 + 12C^3 - C^4}{6C^3}.$$

POD for Coverage $1.5 \le C \le 2$

In Fig. 7 we see that, if we concentrate on the area between searchers 1 and 2, there are again three regions to consider, A, B, and C.

For an object located in region A at a distance x from searcher 1 we have $0 \le x \le (2n-1)W$, and

$$P(x) = 1 - \widetilde{p_0}(x)\widetilde{p_1}(x)\widetilde{p_2}(x)\widetilde{p_3}(x) = 1 - \frac{nW+x}{W}\frac{x}{W}\frac{nW-x}{W} = \frac{W^3 - xn^2W^2 + x^3}{W^3}.$$

Thus, the probability of seeing an object located anywhere in region A is

$$P_A = \frac{1}{W^3}\int_0^{(2n-1)W}(W^3 - xn^2W^2 + x^3)\,dx = 2Wn^4 - \frac{3}{4}W - 6Wn^3 + \frac{11}{2}Wn^2$$

For an object located in region B at a distance x from searcher 1 we have $(2n-1)W \le x \le (1-n)W$, and

$$P(x) = 1-\widetilde{p_0}(x)\widetilde{p_1}(x)\widetilde{p_2}(x)\widetilde{p_3}(x) = 1-\frac{nW+x}{W}\frac{x}{W}\frac{nW-x}{W}\frac{2nW-x}{W} = \frac{W^4 - 2xn^3W^3 + x^2n^2W^2 + 2x^3nW - x^4}{W^4}.$$

Thus, the probability of seeing an object located anywhere in region B is

$$P_B = \frac{1}{W^4}\int_{(2n-1)W}^{(1-n)W}(W^4 - 2xn^3W^3 + x^2n^2W^2 + 2x^3nW - x^4)\,dx = \frac{8}{5}W - \frac{9}{10}Wn^5 + 6Wn^3 - \frac{22}{3}Wn^2.$$

For an object located in region C at a distance x from searcher 1 we have $(1-n)W \le x \le nW$, and

$$P(x) = 1 - \widetilde{p_0}(x)\widetilde{p_1}(x)\widetilde{p_2}(x)\widetilde{p_3}(x) = 1 - \frac{x}{W}\frac{nW-x}{W}\frac{2nW-x}{W} = \frac{W^3 - 2xn^2W^2 + 3x^2nW - x^3}{W^3}.$$

Thus, the probability of seeing an object located anywhere in region C is

$$P_C = \frac{1}{W^3}\int_{(1-n)W}^{nW}(W^3 - 2xn^2W^2 + 3x^2nW - x^3)\,dx = 2Wn^4 - \frac{3}{4}W - 6Wn^3 + \frac{11}{2}Wn^2.$$

Notice that $P_A = P_C$, as expected by symmetry.

Thus, the probability of detection for an object located anywhere between searcher 1 and searcher 2 in Fig. 7 is

$$POD = \frac{P_A + P_B + P_C}{nW} = \frac{2\left(2Wn^4 - \frac{3}{4}W - 6Wn^3 + \frac{11}{2}Wn^2\right) + \left(\frac{8}{5}W - \frac{9}{10}Wn^5 + 6Wn^3 - \frac{22}{3}Wn^2\right)}{nW},$$

which reduces to

$$POD = \frac{3 + 110n^2 - 180n^3 + 120n^4 - 27n^5}{30n} = \frac{-27 + 120C - 180C^2 + 110C^3 + 3C^5}{30C^4}.$$

Putting these together we find

$$POD(C) = \begin{cases} C & \text{if} \quad 0 < C \le 0.5, \\ \frac{1-6C+12C^2-2C^3}{6C^2} & \text{if} \quad 0.5 \le C \le 1, \\ \frac{-3+9C-12C^2+12C^3-C^4}{6C^3} & \text{if} \quad 1 \le C \le 1.5, \\ \frac{-27+120C-180C^2+110C^3+3C^5}{30C^4} & \text{if} \quad 1.5 \le C \le 2. \end{cases}$$

The end values ($C = 0.5, 1$, and 1.5) once again provide a useful check.

Figure 8 shows these PODs as a function of coverage for the Definite Range and Linear LRCs.

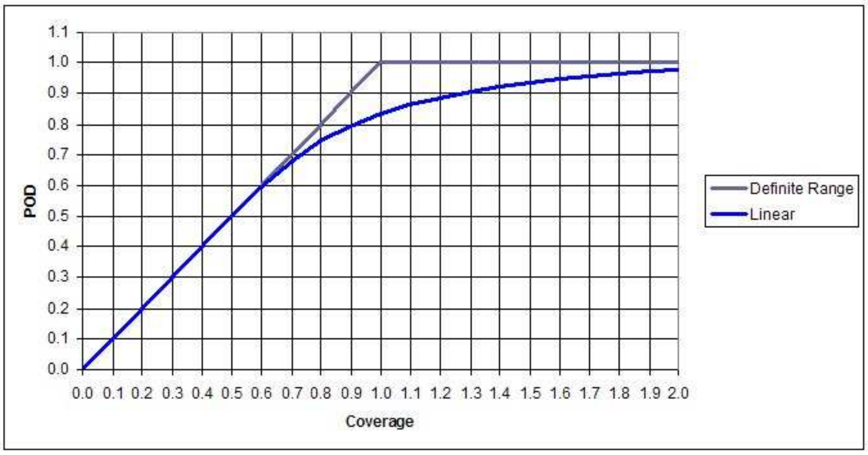

Figure 8: POD versus Coverage for grid search

Summary

This paper has demonstrated a number of points which may help to provide an understanding of the application of search theory to land SAR.

1. A lateral range curve (LRC) describes the detection characteristics of a single sensor; there will be as many LRCs as there are combinations of sensor, search object, terrain, vegetation and visibility, that is, a very large number.
2. The probability given by a LRC relates to an object at a certain distance from the searcher's path; the accepted meaning of the term probability in POD relates to an object that is within a certain distance of the searcher's path.
3. The effective sweep width is the area under the LRC.
4. When a field team grid searches, the POD is influenced by the fact that the LRCs can overlap; the degree of overlapping, and therefore the POD, depends on the spacing between adjacent searchers.
5. Their coverage of the sector that they are searching is determined by their spacing.
6. Graphs of POD against coverage that are derived from overlapping LRCs assume that the spacing between each pair of adjacent searchers is the same and remains the same; if they are not then the graph cannot be used.

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